# Sensor Deception Detection and Radio-Frequency Emitter Localization

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- Maritime Effects of GPS Spoofing
- Ship and Spoofer Model
- Results

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- Single-Emitter Localization Algorithms
- Experimental Work

### Conclusion

# Secure Control of Autonomous Systems



Security researcher Charlie Miller attempts to extract a Jeep Cherokee from a ditch after its brakes were remotely disabled by a cyber attack in a controlled test (Andy Greenberg/Wired).

# Secure Control of Autonomous Systems



Security researcher Jahshan Bhatti examines the wreckage of an autonomous helicopter after its GPS receiver was captured and manipulated by a field attack in a slightly less-controlled test.

### What are Field Attacks?



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# GPS Spoofing



- GPS spoofing has been demonstrated against
  - (i) an autonomous helicopter,
  - (ii) a phasor measurement unit, and
  - (iii) a 65-meter superyacht.

# GPS Jamming





#### Jersey Jammer Caper Privacy Precludes GPS



# Published Works (Journal) I

- K. B. Deshpande, G. S. Bust, C. R. Clauer, H. Kim, J. E. Macon, T. E. Humphreys, J. A. Bhatti, S. B. Musko, G. Crowley, and A. T. Weatherwax, "Initial GPS scintillation results from CASES receiver at South Pole, Antarctica," *Radio Science*, vol. 47, no. 5, 2012
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- M. Psiaki, B. O'Hanlon, J. Bhatti, D. Shepard, and T. Humphreys, "GPS spoofing detection via dual-receiver correlation of military signals," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 49, no. 4, pp. 2250–2267, 2013
- B. W. O'Hanlon, M. L. Psiaki, T. E. Humphreys, J. A. Bhatti, and D. P. Shepard, "Real-time GPS spoofing detection via correlation of encrypted signals," *Navigation, Journal of the Institute of Navigation*, vol. 60, no. 4, pp. 267–278, 2013
- A. J. Kerns, D. P. Shepard, J. A. Bhatti, and T. E. Humphreys, "Unmanned aircraft capture and control via GPS spoofing," *Journal of Field Robotics*, vol. 31, no. 4, pp. 617–636, 2014

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- ② J. A. Bhatti, T. E. Humphreys, and B. M. Ledvina, "Development and demonstration of a TDOA-based GNSS interference signal localization system," in *Proceedings of the IEEE/ION PLANS Meeting*, April 2012, pp. 1209–1220

- The underlying theme of this dissertation is the detection and localization of GNSS-based field attacks.
- To that end, this dissertation makes two primary contributions:
  - a novel GNSS deception detection technique that operates at the sensor fusion level, and
  - estimation algorithms using Monte-Carlo sampling methods for direct geolocation of radio-frequency emitters.

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## GPS Spoofing Against a Maritime Surface Vessel







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## Modern Integrated Bridge System



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## **Detection Problem**



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- What is the optimal detection test for fixed time-to-detect
  - if  $t_0$  and  $z_m(t)$  are known?
  - if only  $z_{\rm m}(t)$  is known?
  - if only  $t_0$  is known?

## **Detection Problem**



- What are near-optimal detection procedures for unknown t<sub>0</sub> and z<sub>m</sub> (t)? (Willsky, 1976)
- What detection procedures take into account integrity risk? (Joerger and Pervan, 2013; Khanafseh et al., 2014)

# Integrity Risk

• A hazardously misleading information (HMI) event is defined as

$$E = \bigvee_{t \ge t_0} \left( \|z_{\mathrm{m}}(t)\| > L \land \left(\bigwedge_{t_0 < kT_s < t} q(k) < \lambda\right) \right)$$
$$= \bigwedge_{t_0 < kT_s \le t_L} q(k) < \lambda,$$

where

- q(k) is a test statistic that monitors the presence of spoofing,
- $t_0$  is the start time of the spoofing attack, and
- $t_L$  is the first time hazardous conditions are encountered.
- The mean integrity risk of the detection framework is given by

$$I_R = \int_0^1 P\left(E|t_0 = \beta T_s\right) d\beta.$$

- The optimal sampling time  $T_s^{\star}$  minimizes the worst-case integrity risk  $I_R^{\star}$ .
- $T_s^{\star}$  and  $I_R^{\star}$  are the solution to the optimization problem given by

 $\begin{array}{ll} \min_{T_s} \max_{v_{\max}} & I_R \\ \text{s.t.} & a \leq v_{\max} \leq b \end{array}$ 

The continuous-time ship dynamics model is given by

$$\dot{\eta}(t) = A\eta(t) + Bu(t) + E\tilde{v}(t),$$

where

$$\eta = \begin{bmatrix} x & y & d_x & d_y \end{bmatrix}^{\mathrm{T}} \text{ is the state vector,} \\ A = \begin{bmatrix} 0 & I \\ 0 & -\frac{1}{T_d}I \end{bmatrix}, B = \begin{bmatrix} I \\ 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ I \end{bmatrix}, \\ u = U \begin{bmatrix} \sin \psi & \cos \psi \end{bmatrix}^{\mathrm{T}} \text{ is the control, and} \\ \tilde{v} = \begin{bmatrix} v_x & v_y \end{bmatrix}^{\mathrm{T}} \text{ is AWGN with intensity } Q_c = \sigma_d^2 I.$$

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### The GPS measurement model for sampling time $T_s$ is given by

$$z(k) = H\eta(kT_s) - z_{\rm m}(kT_s) + w(k),$$

where

$$z_{\rm m}(t)$$
 is the spoofer-induced modulation,  
 $H = \begin{bmatrix} I & 0 \end{bmatrix},$   
 $w(k) \sim \mathcal{N}(0, R),$  and  
 $R = \sigma_p^2 I.$ 

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### Estimation Error

- The optimal sequential estimator for the system model under deception-free conditions (i.e.  $\forall t \ z_{\rm m}(t) = 0$ ) is the Kalman filter.
- The *a posteriori* estimation error and innovation are defined as

$$\hat{\epsilon}(k) \triangleq \eta(kT_s) - \hat{\eta}(k)$$
$$\nu(k) \triangleq z(k) - H\bar{\eta}(k).$$

• The expected value of the estimation error and innovation are given by (in steady state)

$$\mathbb{E}\left[\hat{\epsilon}(k)\right] = (I - KH)F\mathbb{E}\left[\hat{\epsilon}(k)\right] - Kz_{\mathrm{m}}\left(kT_{s}\right)$$
$$\mathbb{E}\left[\nu\left(k\right)\right] = HF\mathbb{E}\left[\hat{\epsilon}\left(k\right)\right] - z_{\mathrm{m}}\left(kT_{s}\right),$$

where  $F = e^{AT_s}$  and K is the Kalman gain.

• The estimation error and innovation are clearly biased under a spoofing attack.

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• For an arbitrary spoofing profile, the optimal statistic (for a generalized likelihood ratio test) has the form

$$q(k) = \nu(k)^T S^{-1}\nu(k) \sim \chi^2 \left( n_z, \mathbb{E} \left[ \nu(k) \right]^T S^{-1} \mathbb{E} \left[ \nu(k) \right] \right),$$

and is typically used for innovations-based fault detection.

• No claims of optimality of the normalized innovation squared (NIS) detection statistic with respect to the current framework are made.

## Integrity risk $I_R$ vs. sampling time $T_s$



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## Minimax integrity risk $I_R^{\star}$ vs. $\overline{L}$ and $M_F$



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## Experimental Results





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- Some open questions remain:
  - Can the framework be applied to an inertial measurement unit or clock model, which both have drift parameters governed by Gauss-Markov processes?
  - Can the integrity risk optimization problem be recast so that the detection statistic and attack profile are free parameters?

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### **1** Conclusion

#### Table : Strengths and weaknesses of different measurement types.

	RSS	AOA	T/FDOA
Antenna and RX complexity	Low	High	Medium
Localization accuracy	Low	Medium	High
Time synchronization	ms	ms	ns
Network throughput	Low	Low	High

## Two-Step Emitter Geolocation

• Traditional two-step geolocation approach (Stein, 1981).





(Sidi and Weiss, 2014)

- The dissertation extends the prior art in the following ways:
  - Theory
    - Relaxed constant Doppler assumption.
    - Developed hybrid Monte-Carlo sampling Kalman filter.
    - Used path-constrained dynamic models.
  - Practice
    - Conducted three experiments.
    - Elucidated important implementation details.
    - Used CDGNSS to extend coherent integration time.

Table : Description of three different types of emitter dynamics models.

Type	State Space	Time History
NS	$\eta = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$	$x(t) = x_0$ $y(t) = y_0$
NCV	$\eta = egin{bmatrix} x_0 \ \dot{x} \ y_0 \ \dot{y} \end{bmatrix}$	$x(t) = x_0 + \dot{x}t$ $y(t) = y_0 + \dot{y}t$
NCVP	$\eta = \begin{bmatrix} s_0 \\ \dot{s} \end{bmatrix}$	$x(t) = T_x (s_0 + \dot{s}t)$ $y(t) = T_y (s_0 + \dot{s}t)$

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Table : Description of three different types of emitter dynamics models.

Type	Update	Matrix Definitions		
NS	$\eta\left(k+1\right) = F_{0}\eta\left(k\right) + v\left(k\right)$	$F_0 = I_{2 \times 2}$		
	$v\left(k ight)\sim\mathcal{N}\left(0,Q_{0} ight)$	$Q_0 = q_0 I_{2 \times 2} T$		
NCV	$\eta\left(k+1\right) = F_{1}\eta\left(k\right) + v\left(k\right)$	$F_1 = \begin{bmatrix} F_2 & 0\\ 0 & F_2 \end{bmatrix}$		
	$v\left(k\right)\sim\mathcal{N}\left(0,Q_{1} ight)$	$Q_1 = q_1 \begin{bmatrix} Q_{cv} & 0\\ 0 & Q_{cv} \end{bmatrix}$		
NCVP	$\eta (k+1) = F_2 \eta (k) + v (k)$ $v (k) \sim \mathcal{N} (0, Q_2)$	$F_2 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$ $Q_2 = q_2 Q_{cv}$ $Q_2 = \begin{bmatrix} \frac{1}{3}T^3 & \frac{1}{2}T^2 \end{bmatrix}$		
		$\begin{bmatrix} Q_{cv} - \lfloor \frac{1}{2}T^2 & T \end{bmatrix}$		

• Consider the simplified signal model for the ith receiver

$$r_i = \alpha_i H_i\left(\eta\right) s + n_i,$$

where s and  $n_i \sim C\mathcal{N}(0, \sigma_n^2 I)$  are the complex baseband emitter signal and noise vectors, respectively,  $\alpha_i$  is the complex path attenuation, and  $H_i(\eta)$  is a complex matrix that time and phase shifts the signal vector.

• The likelihood function is given by

$$L''(\eta, s, \alpha | z) \propto \exp\left(-\frac{1}{\sigma_n^2} \sum_{i=1}^{N_r} \|r_i - \alpha_i H_i(\eta) s\|^2\right).$$

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• In passive geolocation,  $\alpha$  and s are nuisance parameters. A reasonable approach is to replace them with their maximum likelihood estimates as in (Sidi and Weiss, 2014), i.e.

$$L(\eta|z) = \max_{s,\alpha} L''(\eta, s, \alpha|z).$$

• After some algebra,

$$L\left(\eta|z\right) \propto \exp\left(\frac{1}{\sigma_{n}^{2}}\lambda_{\max}\left(\bar{D}\left(z,\eta\right)\right)\right),$$

where

$$\bar{D}_{i,j}(z,\eta) = r_i^{\mathrm{H}} H_i(\eta) H_j^{\mathrm{H}}(\eta) r_j.$$

## Generalized Cross-Correlation Function

• Recall the familiar cross-correlated complex ambiguity function (CAF)

$$S'(\tilde{z}_{1}(t), \tilde{z}_{2}(t), \tau_{0}, f_{D}) \triangleq \int_{0}^{T} \tilde{z}_{1}(t) \, \tilde{z}_{2}^{\star}(t + \tau_{0}) \, e^{-j2\pi f_{D}t} dt,$$

where  $\tau_0$  is a constant delay and  $f_D$  is the Doppler frequency. • Now, consider the GCCF for signals  $\tilde{z}_1(t)$  and  $\tilde{z}_2(t)$ 

$$S\left(\tilde{z}_{1}\left(t\right),\tilde{z}_{2}\left(t\right),\tau_{1}\left(t\right),\tau_{2}\left(t\right)\right) \triangleq \int_{0}^{T} \tilde{z}_{1}\left(t+\tau_{1}\left(t\right)\right)\tilde{z}_{2}^{\star}\left(t+\tau_{2}\left(t\right)\right)e^{j2\pi f_{c}[\tau_{1}(t)-\tau_{2}(t)]}dt,$$

where  $\tau_i(t)$  is the delay time history for received signal  $i \in \{1, 2\}$ and T is the length of the integration interval.

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• Naive grid search is appropriate for the 2-D NS model; however,

- the state space is evaluated inefficiently with a fixed grid,
- the search space can become unwieldy for the 4-D NCV model, and
- the dynamical constraint between position and velocity over time in both the NCV and NCVP model is not enforced.

# Hybrid Kalman Filter with Monte-Carlo Sampling

- A Kalman filter approach has two advantages over GS:
  - KF allows smoothing the measurement information with the dynamical constraints, and
  - *a priori* emitter state and covariance can be used constrain the search space to a smaller region.
- Hybrid measurement sampling approach:





- The PF provides the same improvements over GS as KF, but has two advantages over KF:
  - no initial guess required, so that a uniform sampling over state space can be used, and
  - can propagate non-Gaussian pdf for emitter state, which can help prevent divergence due to transient multipath.

## Emitter Localization Post-Processing Workflow



### White Sands Missile Range Experiment





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## Two-Receiver NCVP Estimability



# Three-Receiver NCVP Estimability



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## Four-Receiver NCVP Estimability



### GS and PF Algorithms in NCVP State Space





Time = 10 s

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### GS and PF Algorithms in NCVP State Space





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### GS and PF Algorithms in NCVP State Space



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## PF Performance with Varying Number of Particles



## PF Performance Comparing NCVP and NCV Model



# UAV Experiment

• The UAV experiment was designed to mimic applications where a stationary base and dynamic rover platform work together to locate a target.





Fully-assembled Stereo-based EMLOC sensor

Stereo-based EMLOC sensor with top cover removed

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# UAV Experiment



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## GS with Non-Coherent Averaging





 $T = 5 \,\mathrm{s}$ 

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## GS with Non-Coherent Averaging





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## GS with Non-Coherent Averaging





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- What are the implications of multiple emitters on the estimation architecture in terms of computational and algorithmic complexity?
- How can motion planning algorithms for dynamic receiver platforms based on an information-seeking control law be applied to direct emitter geolocation? (Hoffmann and Tomlin, 2010).

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- A detection framework has been developed to detect spoofing attacks in maritime environments based solely on Doppler log, gyrocompass, and potentially-spoofed GPS measurements.
- The worst-case integrity risk of the detection framework was minimized by optimizing the sampling time of the GPS measurements.
- A passive RF emitter localization system has been developed and analyzed thoroughly with multiple experiments.
- The system employed direct geolocation and long coherent integration techniques, thus improving the system's estimation performance with weak emitters and multipath.

### Conclusion



Security researcher Jahshan Bhatti gazes toward the empty horizon, observing the bent wake caused by the captain steering the yacht to keep the spoofed GPS position, so easily trusted yet so easily manipulated, within a carefully charted corridor.

### PF Performance for UAV Experiment



### UAV on Roof Experiment



 $T=2\,\mathrm{s}, \epsilon=11.5\,\mathrm{cm}$ 



 $T = 5 \, s, \epsilon = 16.7 \, cm$ 

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## UAV on Roof Experiment



 $T=5\,\mathrm{s}, \epsilon=16.7\,\mathrm{cm}$ 



 $T = 10 \, \text{s}, \epsilon = 12.9 \, \text{cm}$ 

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