Opportunistic Frequency Stability Transfer for Extending the Coherence Time of GNSS Receiver Clocks

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The Problem

GNSS signals attenuate 30–50 dB in indoor environments

GNSS receivers can’t acquire or track indoors with a $C/N_0 \approx 7$ dB-Hz
A Possible Solution

Coherently integrate long enough to recover signal power!
How long do we have to wait to acquire?

Rule-of-thumb from detection theory:

$$\text{SNR} = \frac{C}{N_0} \cdot T \geq 14 \text{ dB}$$

for fixed $P_d = 0.95$ and $P_{fa} = 0.001$
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for fixed $P_d = 0.95$ and $P_{fa} = 0.001$
"Now's the time, the time is now"
-Led Zeppelin
What Are Our Options?

1. Carry an atomic clock →
2. Use chip-scale atomic clocks in RX
3. Use small, portable OCXOs

leapsecond.com/pages/atomic-bill
What Are Our Options?

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4. Pull a clock out of thin air
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Oscillator Model

Oscillator generates **sinusoidal output** voltage:

\[ v(t) = \cos(2\pi \nu_0 t + \phi(t)) \]

where \( \nu_0 \) is nominal frequency and \( \phi(t) \) is time-varying phase.

Ideal oscillators are deterministic:

\[ V(t) = \cos(2\pi \nu_0 t) \quad \text{(no offset)} \]
\[ V(t) = \cos(2\pi \nu_0 t + \Phi) \quad \text{(constant offset)} \]
Oscillator Comparison Experiment

- Estimated phase using receiver driven by TCXO and OCXO

- Removed deterministic component due to satellite orbit leaving:
  1. ionospheric errors
  2. ephemeris errors
  3. timing errors in range computation
  4. receiver position errors

- Linear fit removes 2, 3, and 4

- Remaining phase represents phase history of the driving oscillator
TCXO Referenced Carrier Phase Observable

\[ \lambda \phi(t) = r(t) + c [\delta t_R(t) - \delta t_S(t)] + \lambda (\gamma_0 - \psi_0) + \epsilon_{\text{atmo}}(t) + \lambda n_\phi \]
TCXO & OCXO Referenced Carrier Phase Observables

\[ \lambda \phi(t) = r(t) + c [\delta t_R(t) - \delta t_S(t)] + \lambda (\gamma_0 - \psi_0) + \epsilon_{\text{atmo}}(t) + \lambda n_{\phi} \]
OCXO Referenced Carrier Phase Observable

\[ \lambda \phi(t) = r(t) + c \left[ \delta t_R(t) - \delta t_S(t) \right] + \lambda (\gamma_0 - \psi_0) + \epsilon_{\text{atmo}}(t) + \lambda n_\phi \]
TCXO vs. OCXO

![Graph showing SNR vs. time for TCXO and OCXO]

- **SNR [dB]**
  - Acquisition level: 15 dB
  - Tracking level: 0 dB

- **Time [sec]**: From $10^{-1}$ to $10^2$

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**Wireless Networking & Communications Group (WNCG)**

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**Radionavigation Laboratory**
TCXO vs. OCXO

![Graph showing SNR vs. time for TCXO and OCXO]

- Acquisition
- Tracking

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Phasor Rotation
Stable Oscillator

Phasor Rotation
Unstable Oscillator
Frequency Stability Transfer Model

Rotate GNSS phase by the change in aiding signal’s phase relative to local oscillator over \( j \)th intermediate accumulation interval:

\[
\Delta \phi = \left( \frac{f_{\text{GNSS}}}{f_{\text{AID}}} \right) \cdot (\phi_{\text{AID}}[n_j] - \phi_{\text{AID}}[n_{j-1}] + n_\phi)
\]
Frequency Stability Transfer Phasor “Fix-Up”

Phasor Rotation Corrected

$\Delta \hat{\phi}_1$, $\Delta \hat{\phi}_2$, $\Delta \hat{\phi}_3$, $\Delta \hat{\phi}_4$
Synthetic Oscillator Phase History

Create synthetic oscillator via single differencing: \( \Delta \hat{\phi}_{AID} = \phi_{PRN\ 22} \)
Ambient Stable Signals

1. WWVB
   - broadcast from NIST in Colorado
   - designed to synchronize time
   - only stable locally due to ground wave propagation
   - 60 kHz broadcast

2. High-Definition TV (HDTV)
   - available in major metropolitan areas
   - signal is designed to penetrate buildings
   - approx. 700 MHz broadcast

3. Cellular Code Division Multiple Access (CDMA)
Why CDMA?

- CDMA is similar to GPS from receiver standpoint
- Widely available in U.S.
- Base station clocks very stable and synched to GPS
- UHF Band 1930–1990 MHz (tends to attenuate phase noise)
- Dataless pilot channel allows direct computation of phase

Verizon Wireless Coverage Map, 2010
CDMA Laboratory Setup
Frequency Stability Transfer Block Diagram

Despread

Intermediate Accumulation

Rotate

Final Accumulation

Aiding Signal
Code & Carrier
Tracking Loops

$r_{GNSS}[n]$  
$C_{GNSS}[n]e^{j\hat{\Theta}[n]}$

$\Sigma(\cdot)$

$S$

$\Delta\hat{\phi}$

$k_m = \arg\max_k |F[k]|$

$F' = \text{FFT}[S_1', S_2', ..., S_N']$

$S'' = F[k_m]$

$Aiding signal phase change due to local clock$

over $j$th intermediate accumulation interval

$\Delta\hat{\phi}_{CDMA}[j] = \hat{\phi}_{CDMA}[n_j] - \hat{\phi}_{CDMA}[n_{j-1}] + n_{\phi}$
Oscillator Coherence

\[ \langle C_{coh}^2 \rangle \]

- TCXO
- OCXO

Time [sec]
Oscillator Coherence

\[ \left< C^2_{coh} \right> \]

- TCXO
- OCXO
- SYXO

\begin{align*}
\text{time [sec]} & \quad 10^{-1} & \quad 10^0 & \quad 10^1 & \quad 10^2 \\
\text{values} & \quad 1 & \quad 0.75 & \quad 0.5 & \quad 0.25 \\
\end{align*}
Oscillator Coherence

\[ \langle C^2_{\text{coh}} \rangle \]

- TCXO
- OCXO
- SYXO
- CDMA2
Oscillator Coherence

The figure shows the coherence time ($<C^2_{coh}>$) as a function of time (in seconds) for different oscillators:

- TCXO
- OCXO
- SYXO
- CDMA1
- CDMA2

The data is plotted on a log-log scale, indicating the coherence time decreases significantly over time for all oscillators.
Coherence and SNR Relation

\[ \text{SNR}(t) = \langle C_{\text{coh}}^2(t) \rangle \cdot t \cdot (C/N_0) \text{ for fixed } C/N_0 \]
Indoor GNSS tracking and acquisition is possible with commercial GNSS receivers using stable signals of opportunity if you’re willing to hold still for a few seconds!