Precision Limits of Low-Energy GNSS Receivers

Ken Pesyna and Todd Humphreys
The University of Texas at Austin

September 20, 2013

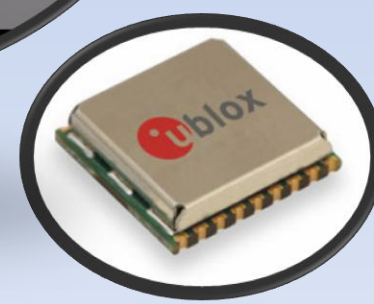
Todd Humphreys

TEDx Austin
x = independently organized TED event

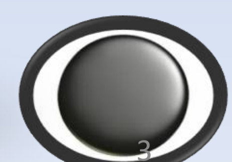
The GPS Dot

Site: www.ted.com
Search: “gps”


WorldTracker GPRS
LOCK
UNLOCK


ublox

??



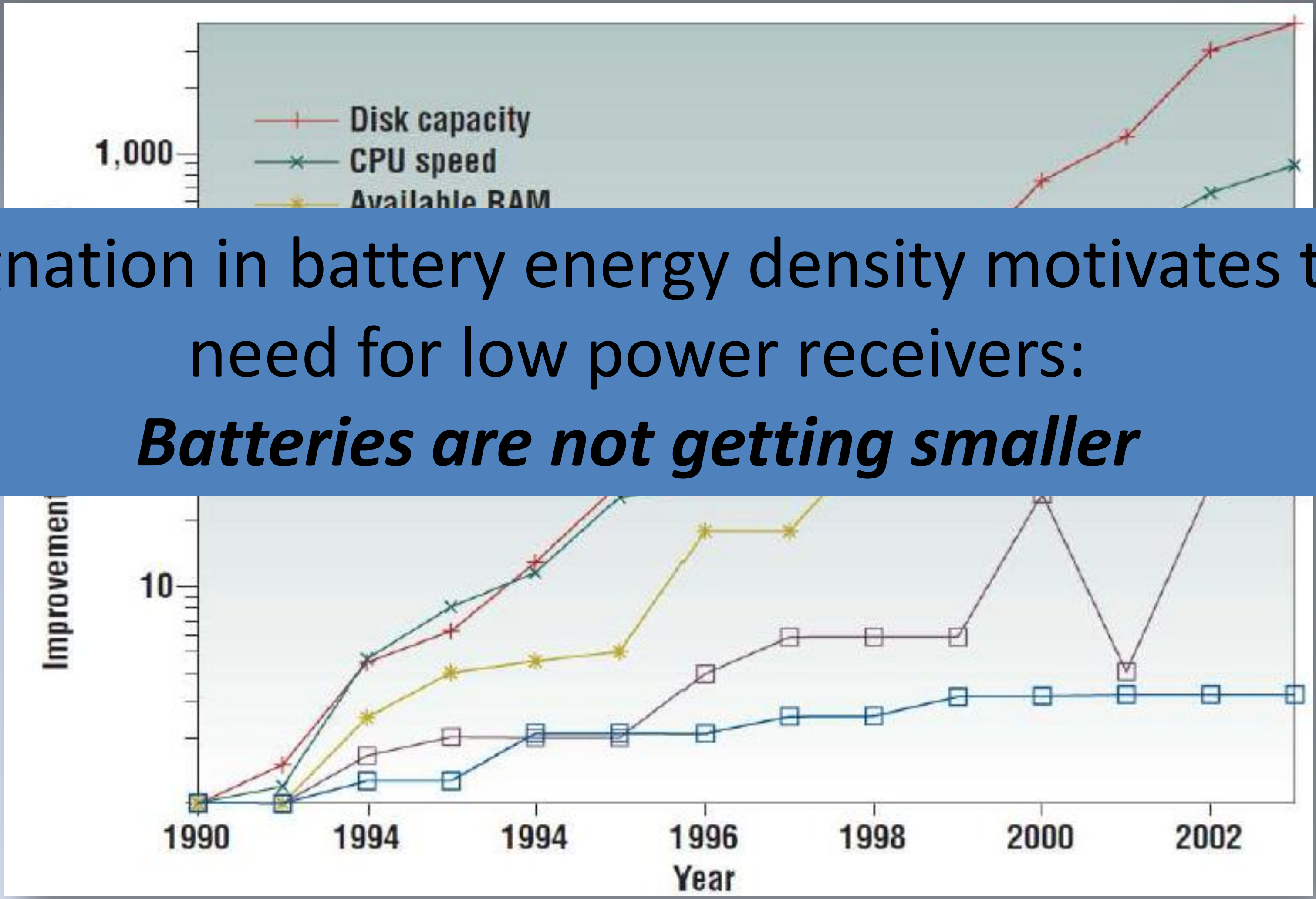
Tradeoffs

- **Size**
- **Weight**
- **Cost**
- **Precision**
- **Power**

Tradeoffs

- ~~Size~~
- ~~Weight~~
- ~~Cost~~
- Precision
- Power

Improvements Since 1990



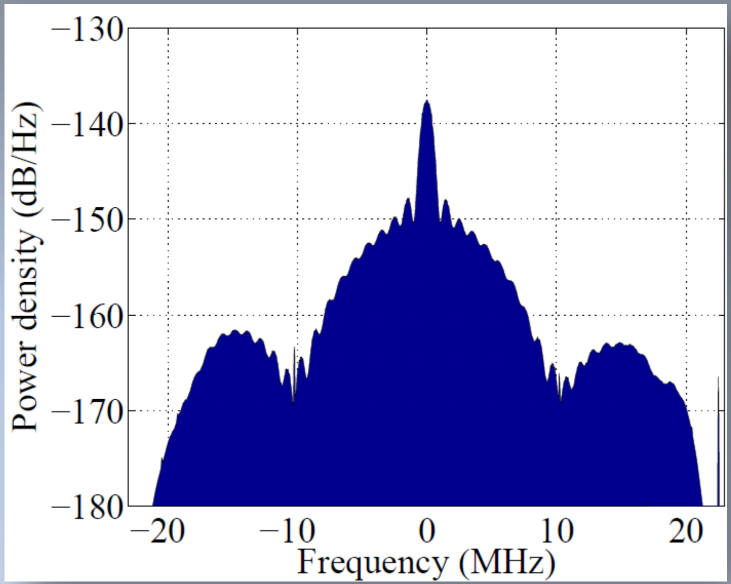
Stagnation in battery energy density motivates the need for low power receivers:

Batteries are not getting smaller

Simple Ways to Save Energy

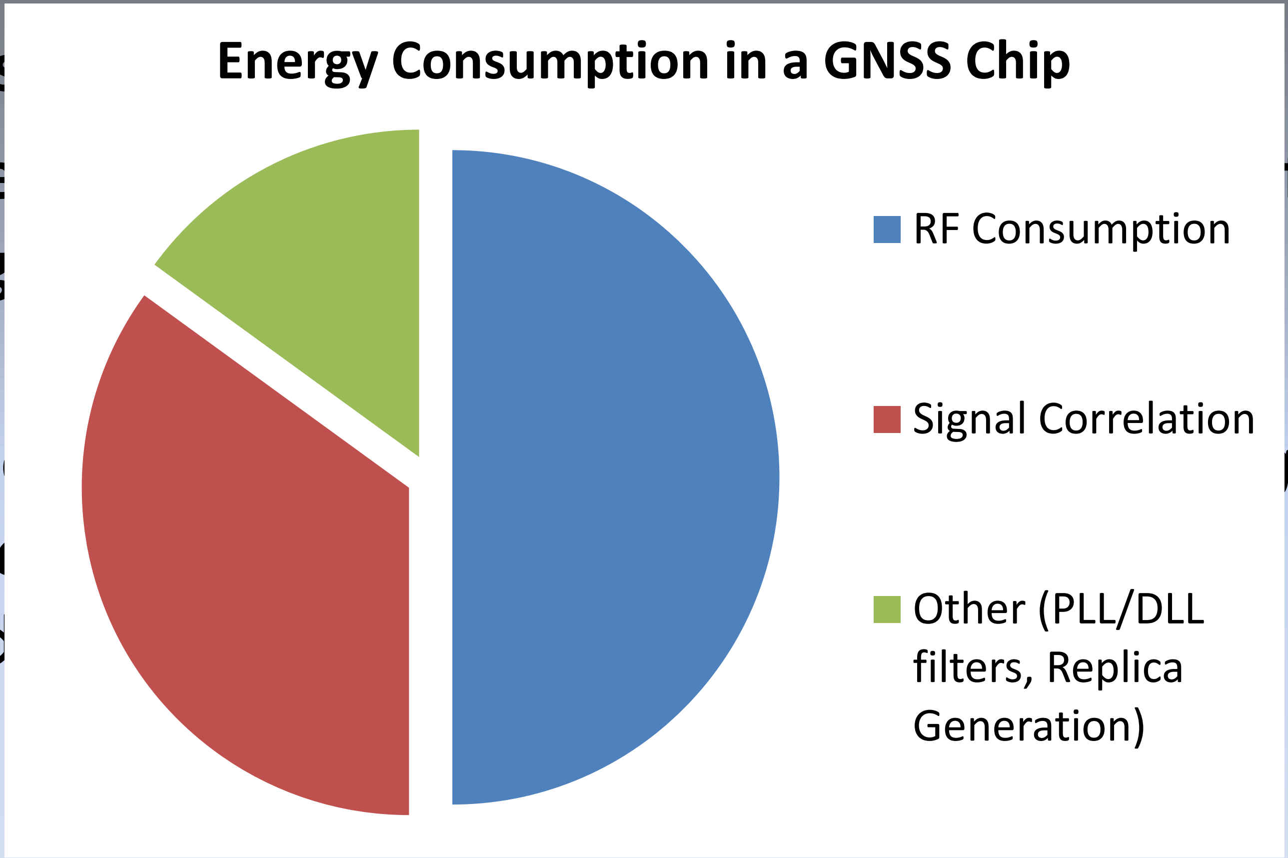
Modify parameters of interest:

1. Track fewer satellites, Nsv
2. Reduce the sampling rate, f_s
3. Reduce integration time, T_{coh}
4. Reduce quantization resolution, N bits



Assumptions

1. Cons
2. Base for roug NSS chip
3. Larg *nal* *corr* prod



Correlation and Accumulation

- **Given $K = f_s * T_{coh}$ samples to correlate**
- **K N-bit multiplies and K-1 $(N+\log_2 K)$-bit adds**
    - **An N-bit add needs N 1-bit full adders**
    - **An N-bit multiply needs (N-1)*N 1-bit full adders**
- **Total number of 1-bit full adders N_A in a CAA**

$$N_A = \underbrace{(K-1)(N+\log_2 K)}_{\text{Accumulation Cost}} + \underbrace{K(N-1)(N)}_{\text{Multiplication Cost}}$$

$$\approx K(N + \log_2 K + N^2 - N) \text{ for large K}$$

$$= K(N^2 + \log_2 K)$$

Energy Required for Correlation and Accumulation

- **Each 1-bit addition takes E_A energy**
- **Energy consumed by a CAA, $E_{CAA} = E_A \cdot N_A$**
- **Total energy consumed by all CAA operations**

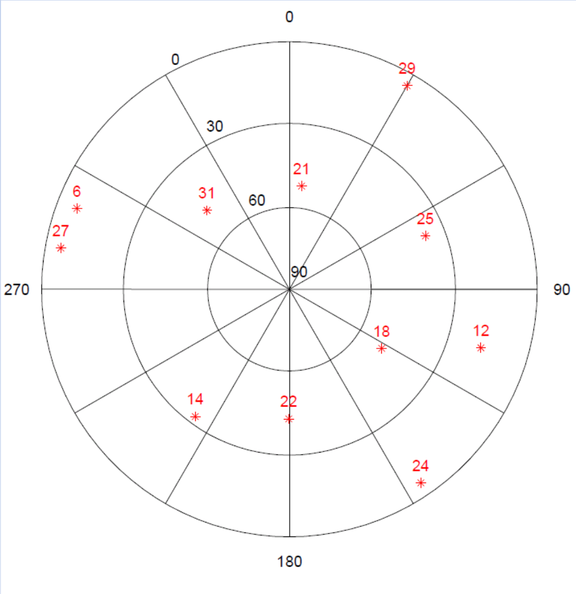
$$E_{\mathrm{Total}} = N_{\mathrm{SV}} * 6 * E_{\mathrm{CAA}}$$

Problem Statement

- **Given a fixed amount "E_{Total}" of Joules, what choices of T_{coh} , f_s , N, and N_{sv} should we make to maximize positioning precision?**

Step 1: Positioning Precision

- **Positioning precision can be characterized by the RMS position-time error σ_{xyzt}**
- **Geometric Dilution of Precision (GDOP) relates σ_{xyzt} to the pseudorange error σ_u:** $\sigma_{xyzt} = \sigma_u \cdot \mathrm{GDOP}$
- $\mathrm{GDOP} = \sqrt{\mathrm{Tr}\,(G^T G)^{-1}}$
- **Optimal geometry:**
- **GDOP_{MIN} =** $\sqrt{\frac{10}{N_{\mathrm{SV}}}}$ **[Zhang 2009]**



Step 2: Code Tracking Error

- Lower bound on coherent early-late discriminator design [Betz 09]:

$$\sigma_{u,\mathrm{EL}}^2 = \frac{\int_{-f_s/2}^{f_s/2} G_s(f)\sin^2(\pi f \Delta)df}{2(2\pi)^2 T_{\mathrm{coh}} \frac{C_s}{N_0}\left(\int_{-f_s/2}^{f_s/2} f G_s(f)\sin(\pi f \Delta)df\right)^2}$$

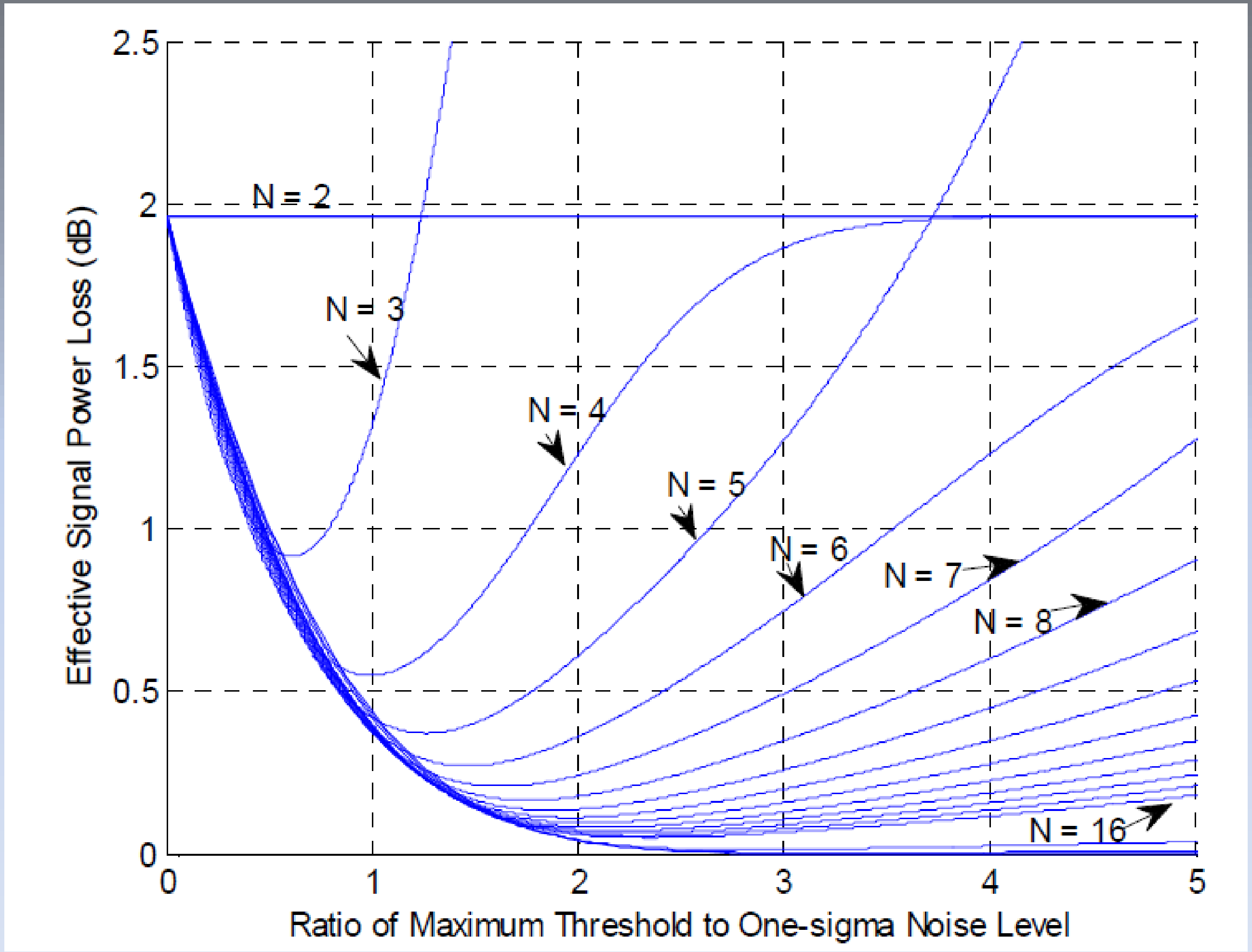
- As the early-late spacing $\Delta \rightarrow 0$, $\sigma_{u,EL} = \sigma_{u,CRLB}$

Step 3: Effective Carrier-to-Noise Ratio

- **The C_s/N_0 is affected by the ADC quantization resolution N**
- **[Hegarty 2011] and others have shown that quantization resolution "N" decreases the overall signal power, C_s**
- **The effective signal power at the output of the correlator $C_{eff} = C_s/Lc$**

Function of Quantization Precision "N"

Effective Carrier-to-Noise Ratio



*Hegarty, ION GNSS 2010, Portland, OR

Quick Recap

- **4 parameters of interest: f_s , N_{sv} , T_{coh} , N**
- **Derived baseband energy consumption:**

$$E_{\mathrm{CAA}} = E_{\mathrm{A}} \cdot (N^2 + \log_2[f_s \cdot T_{\mathrm{coh}}]) \cdot f_s \cdot T_{\mathrm{coh}}$$

$$E_{\mathrm{Total}} = N_{\mathrm{SV}} * 6 * E_{\mathrm{CAA}}$$

- **Derived lower bound on positioning precision:**

$$\sigma_{\mathrm{xyzt}} = \sigma_{\mathrm{u,EL}} \cdot \mathrm{GDOP}$$

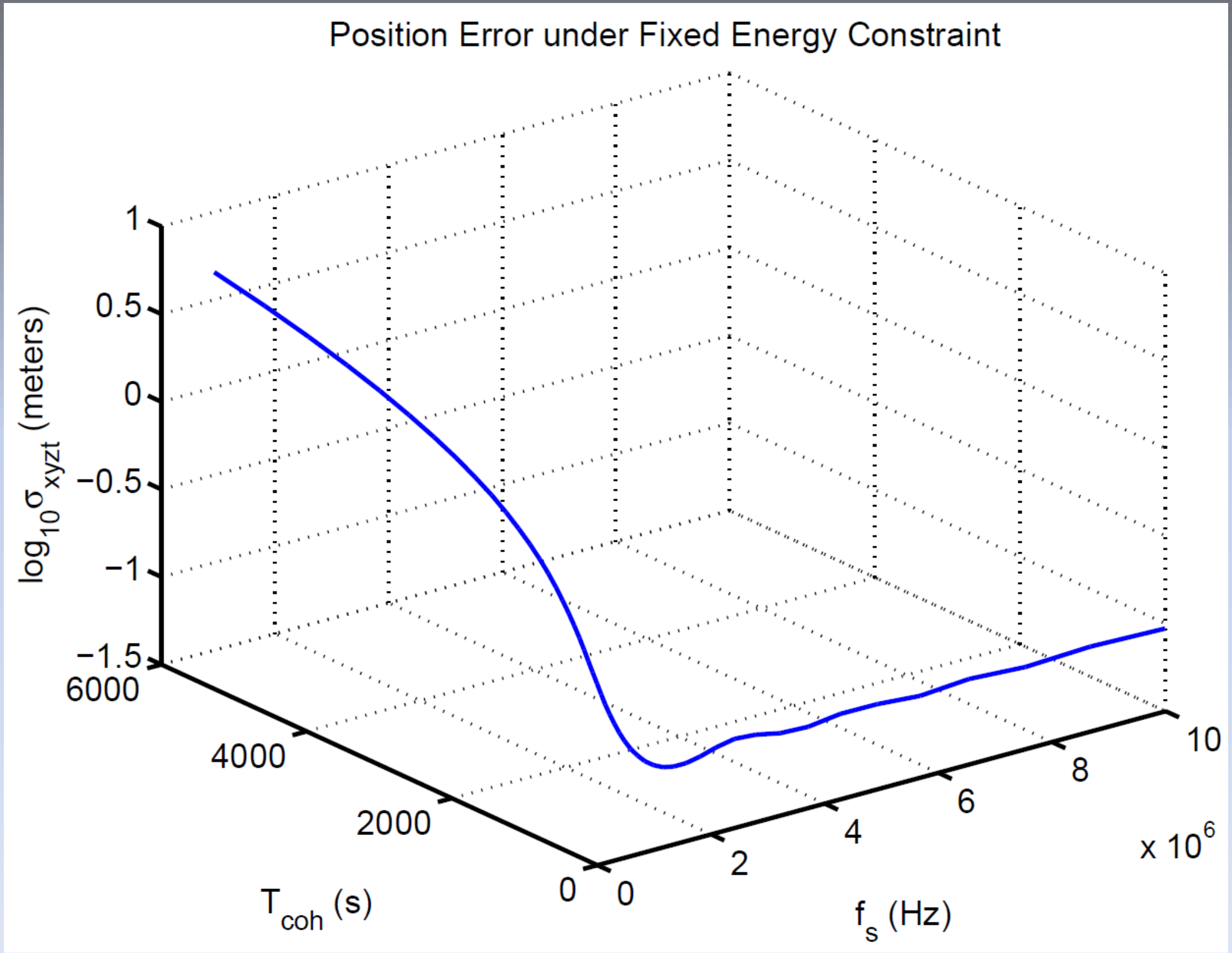
$$\sigma^2_{\mathrm{u,EL}} \rightarrow \frac{1}{4T_{\mathrm{coh}} \frac{1}{T_{\mathrm{c}}} \frac{C_{\mathrm{eff}}}{N_0} \left(f_s - \frac{\sin(f_s \pi T_{\mathrm{c}})}{\pi T_{\mathrm{c}}} \right)} \text{ as } \Delta \rightarrow 0$$

Optimization Problem

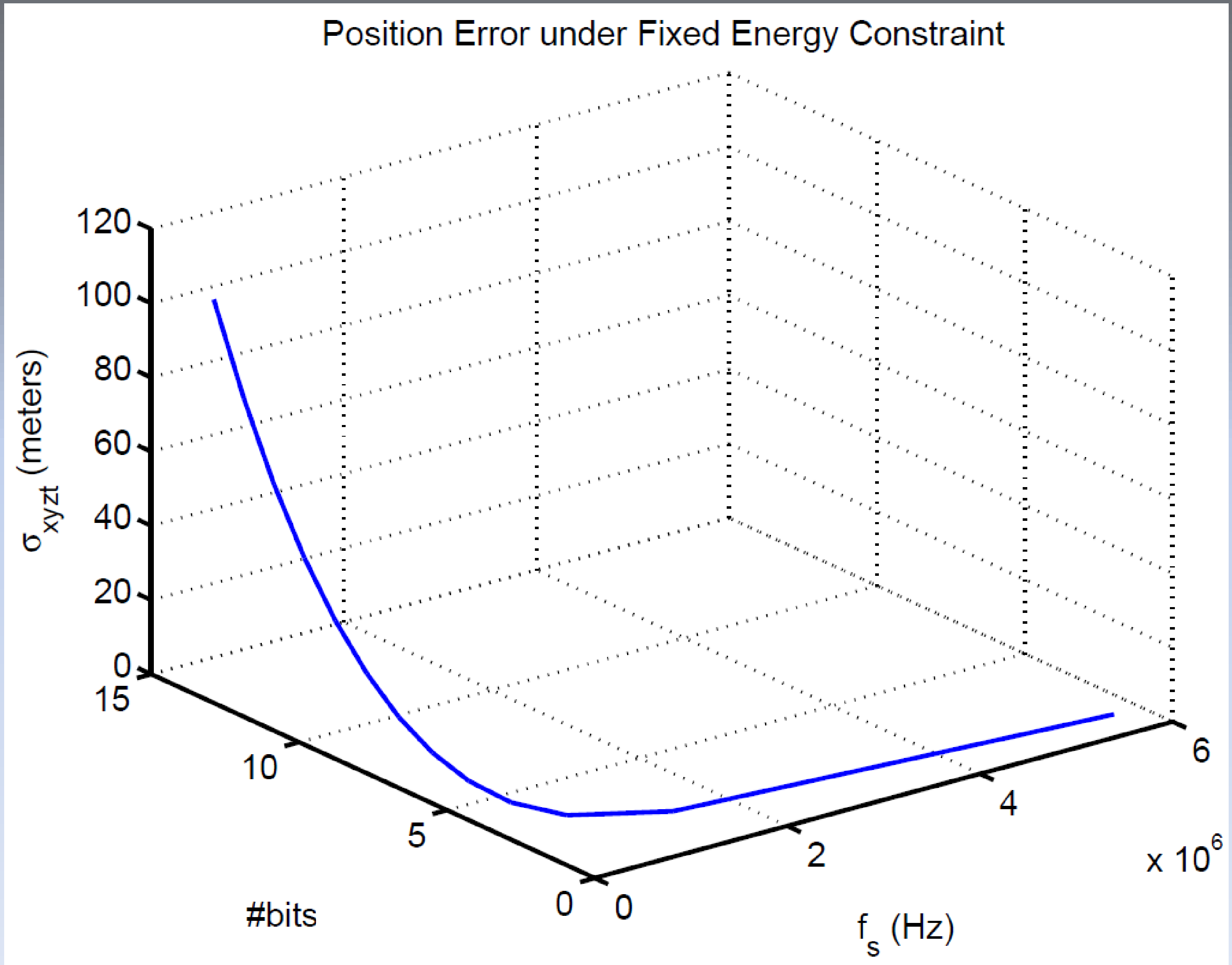
- **We have set up a constrained optimization problem to minimize σ_{xyzt} for a given E_{Total}**

$$\begin{aligned} \underset{N_{SV}, N, f_s, T_{\text{coh}}}{\text{minimize}} \quad & \sigma_{xyzt}(N_{SV}, N, f_s, T_{\text{coh}}) \\ \text{subject to} \quad & E_{\text{Total}} \leq \beta \end{aligned}$$

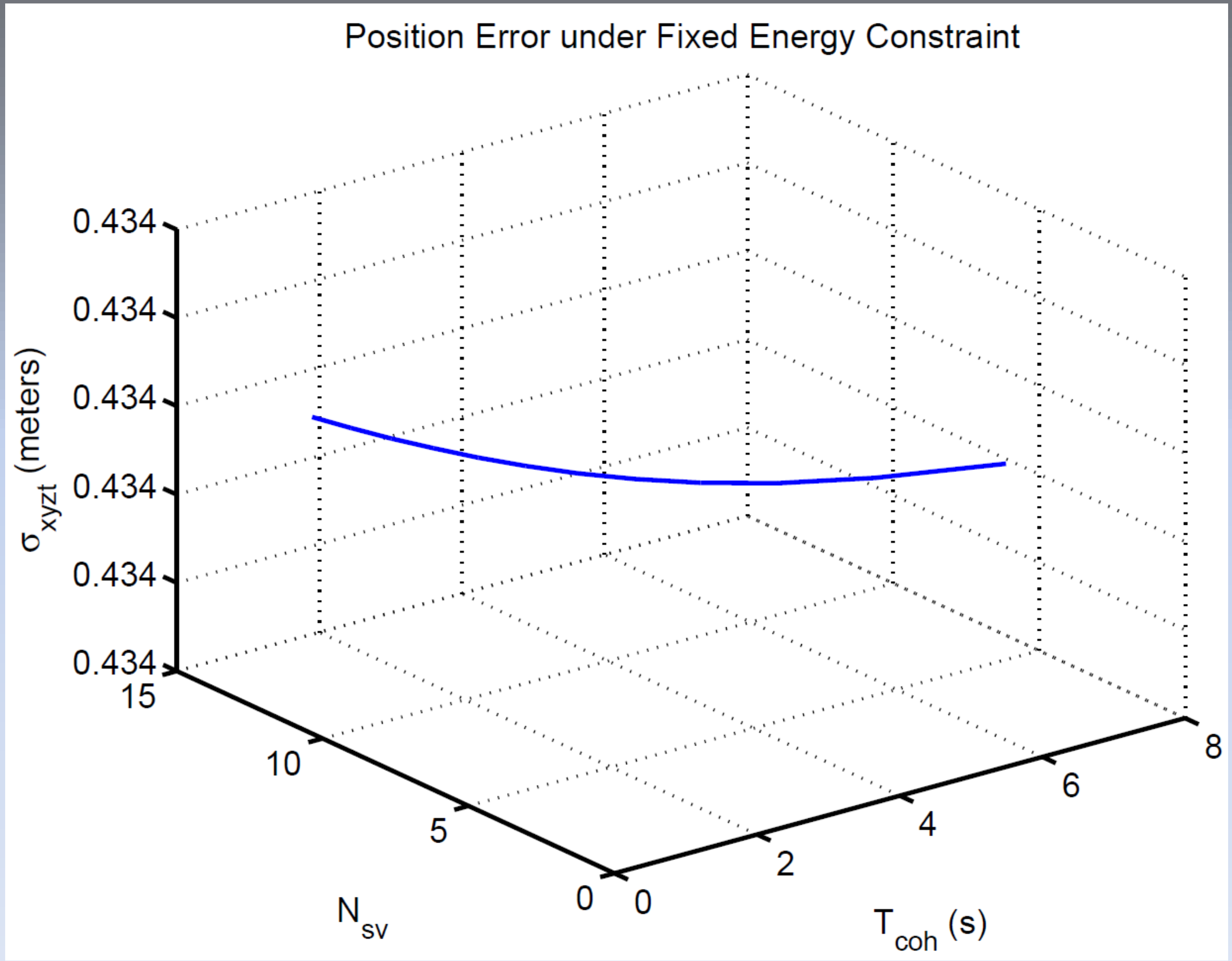
Tradeoff 1: Sampling Rate, f_s vs Integration Time, T_{coh}



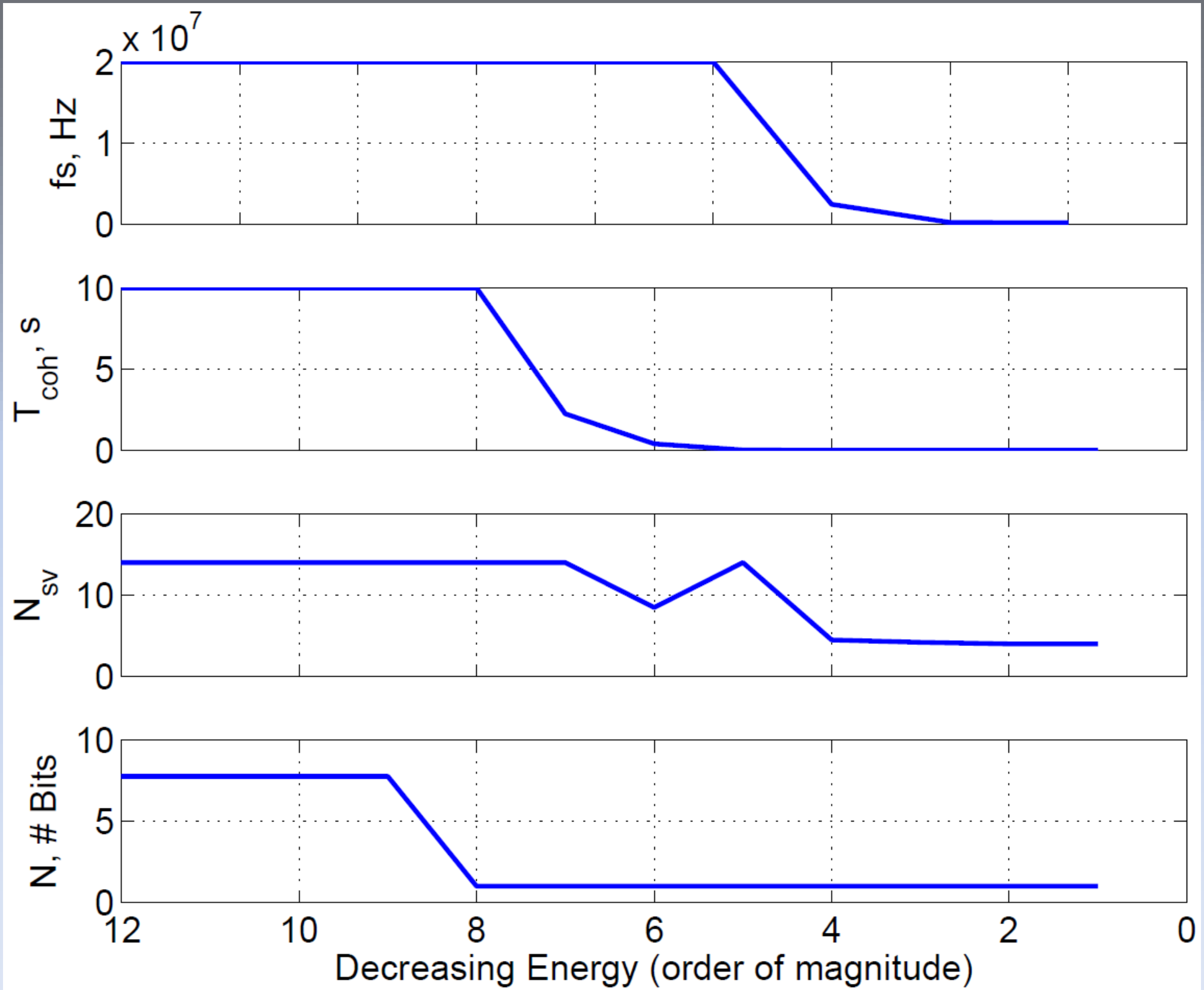
Tradeoff 2: Sampling Rate, f_s vs Quantization Resolution, N



Tradeoff 3: Number of SVs, N_{sv} vs Integration Time, T_{coh}



Optimization Solution versus Declining Energy



Conclusions

1. Investigated how certain parameters relate to energy consumption and positioning precision
2. Posed an optimization problem that solves for optimal values of the 4 parameters of interest under an energy constraint
3. Showed that the industry has come to anticipate many of the same conclusions

THE UNIVERSITY OF TEXAS AT AUSTIN

RADIONAVIGATION LABORATORY

http://radionavlab.ae.utexas.edu